Self-similarity of magnetic field energy density fluctuations in the solar wind: evidence for coronal signature?

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Introduction

- Presence of inertial range and multi-fractal scaling in velocity and magnetic field components are well known [1]
- Magnetic field component spectra show "***1/f***" region, associated with coronal driving [2]
- Fluctuations in magnetic field energy density, $\boldsymbol{\delta(B^2)}$, show an approximate mono-scaling of the power spectrum spanning "***1/f***" region and inertial range
- Scaling properties are solar cycle dependent, suggesting their solar origin

[1] Bruno & Carbone, Living Review 4 (2005);
[2] Matthaeus & Goldstein, PRL (1986)

Choice of observable, data sets

- Fluctuations ***δ(B²)*** are not linear Alfvénic fluctuations
- Magnetic field energy dominates coronal dynamics
- Important for pressure balance structures in the solar wind

ACE:

- 96s averaged MFI data for year 2000, solar maximum
- Contains ~4 x 10^5 samples

WIND:

- 60s averaged MAG measurements
- Solar minimum: year 1996, solar maximum: year 2000
- Both with 4.5 x 10^5 samples

Fluctuations on scale τ: $\delta[B^2](t,\tau) = B^2(t+\tau) - B^2(t)$

Scaling, conditioning

Generalized Structure Functions: $S_m(\tau) = \left\langle \left| \delta B^2 \right|^m \right\rangle \propto \tau^{\zeta(m)}$

Real data includes extreme events: conditioning [3,4]

$$<|\delta B^2|^m> = \int_{-A}^{A} |\delta B^2|^m \, P(\delta B^2, \tau) d(\delta B^2)$$

- Iteratively remove *k-th* largest fluctuation from $\boldsymbol{\delta B^2}$
- Re-compute scaling exponents $\zeta(m)$
- Check the rate of conversion $\zeta(m) \rightarrow \alpha m$

Sensitive discriminator of fractality

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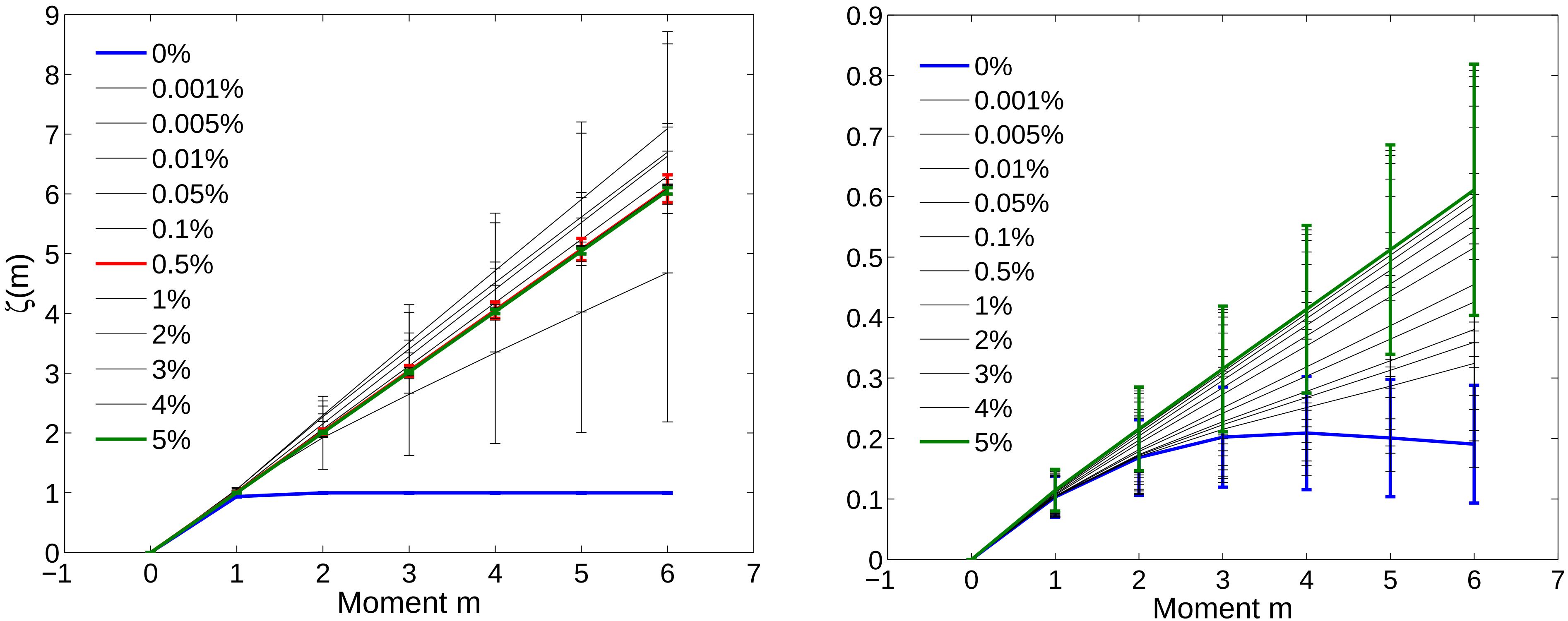
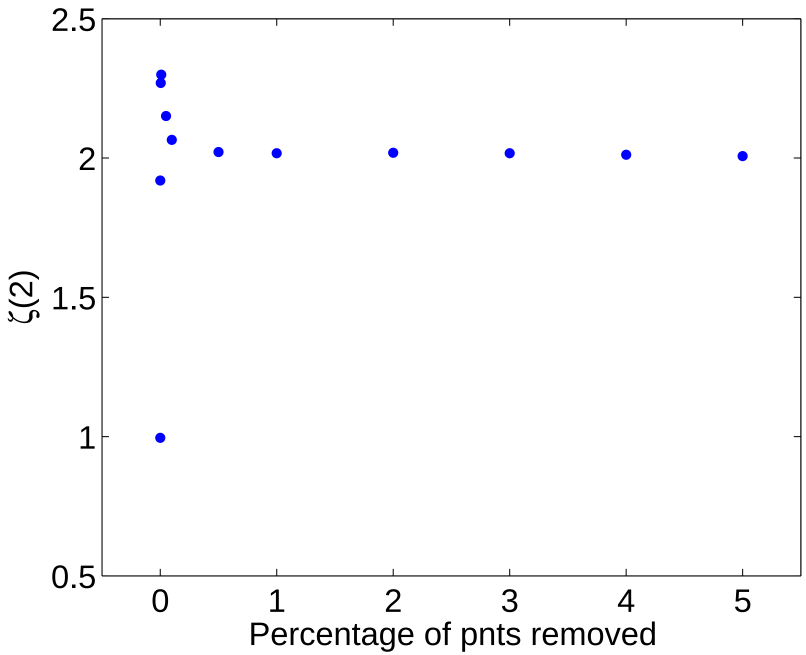
[3] Kiyani *et. al.*, PRL 98 (2007);
[4] Hnat *et. al.*, GRL 34 (2007)

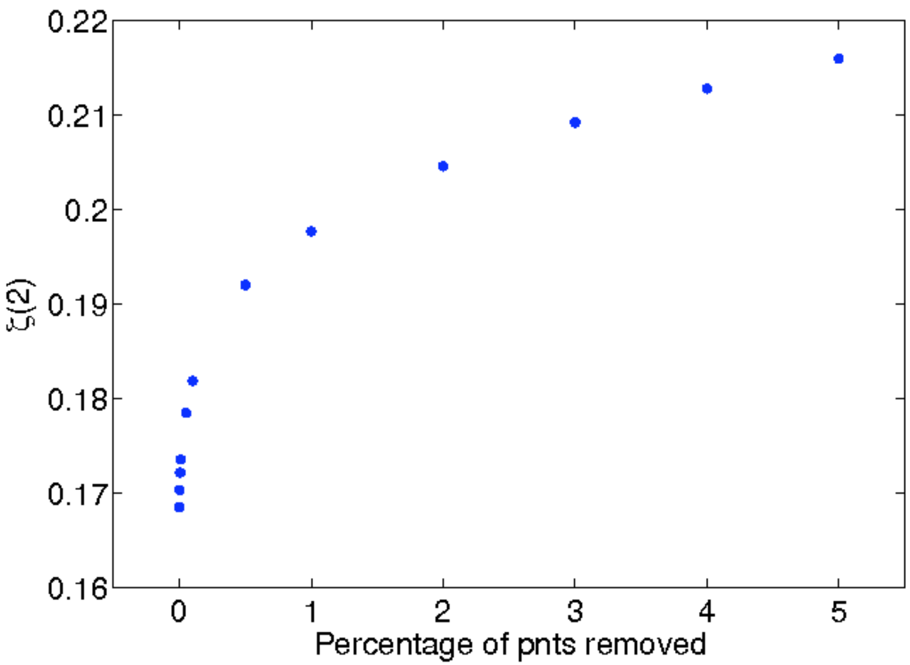
Example: Levy walk, p-model



Scaling of the Levy walk rapidly converges towards a linear trend (after 0.5% outlayers has been removed), but p-model does not converge to a constant linear trend.

Example: Levy walk, p-model

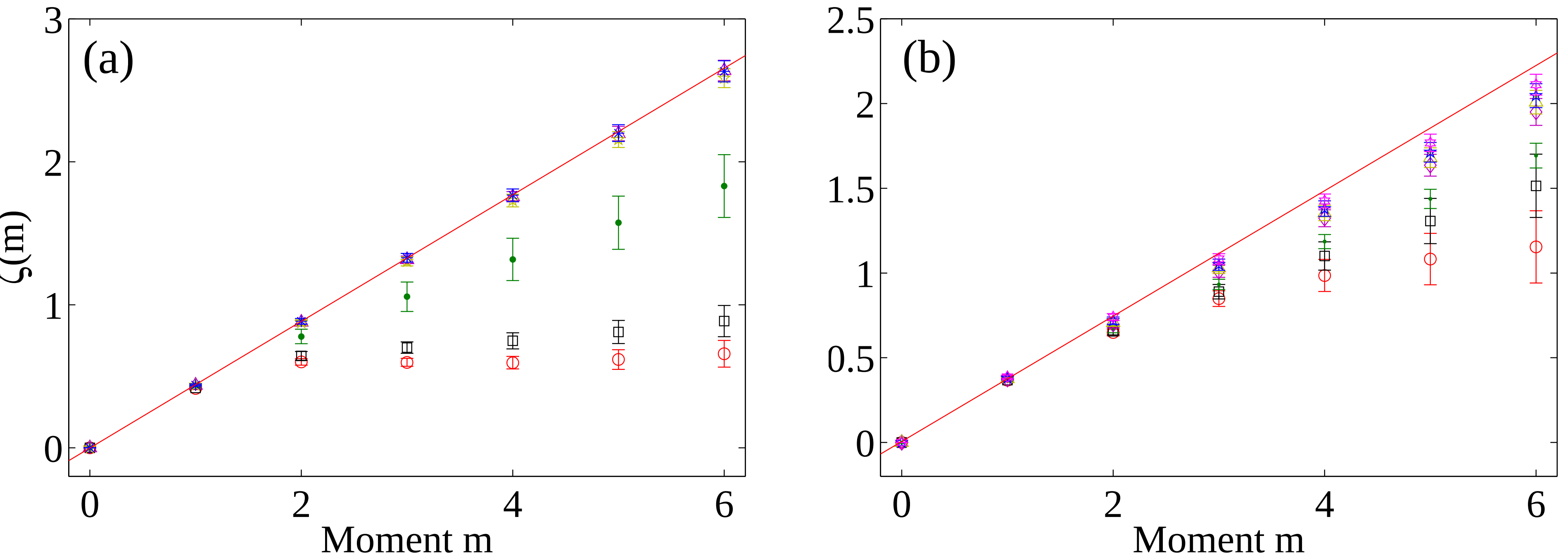




A single moment scaling exponent also converges to a single value for a Levy, but does not show any saturation for p-model.

Self-similarity of δB^2 at solar max.

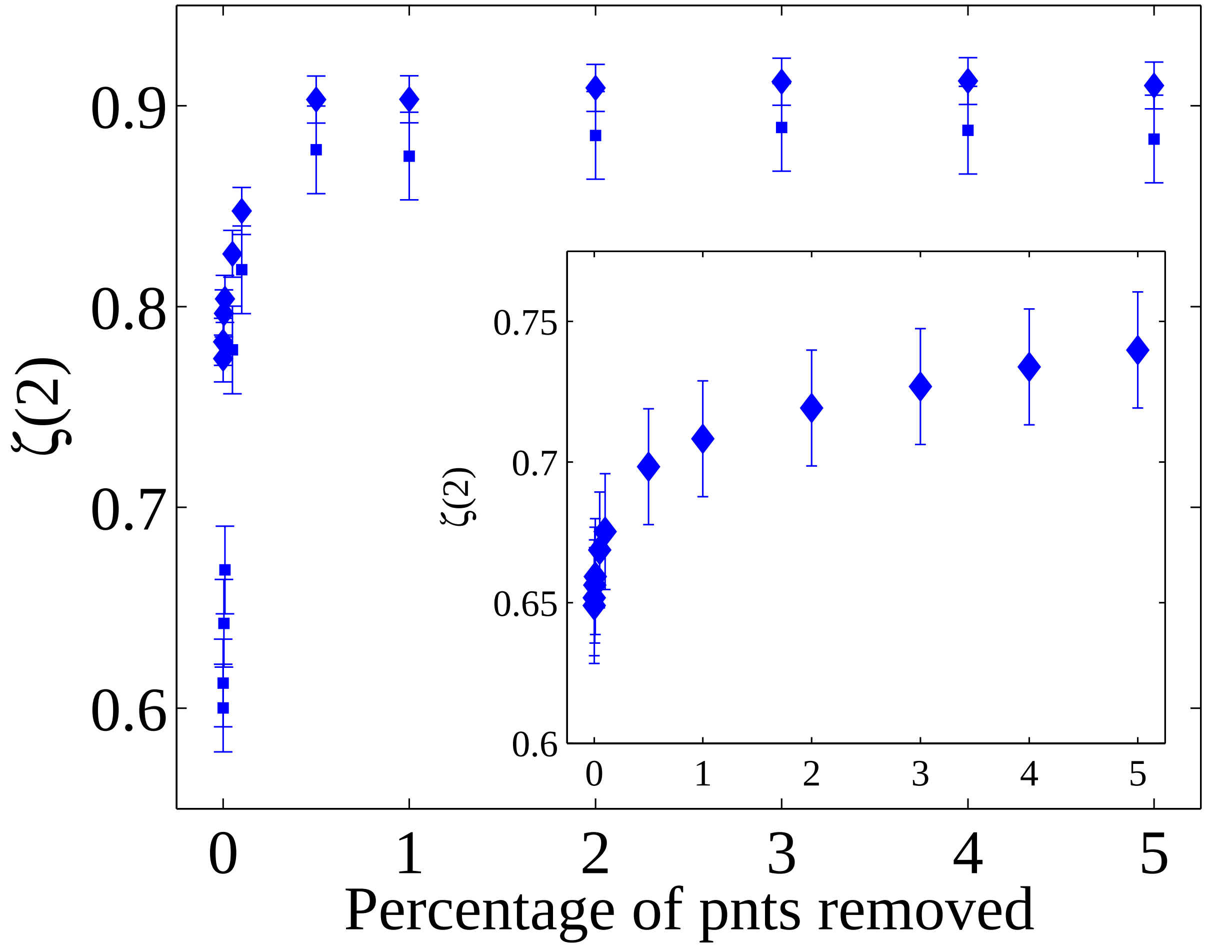
- Convergence of $\zeta(m) \rightarrow \alpha m$ indicates self-similarity



Solar max. fluctuations show linear scaling after 0.5% outlayers are removed, no linear trend for solar min.

Self-similarity of δB^2 at solar max.

- Convergence of *ζ(2)* to a unique value

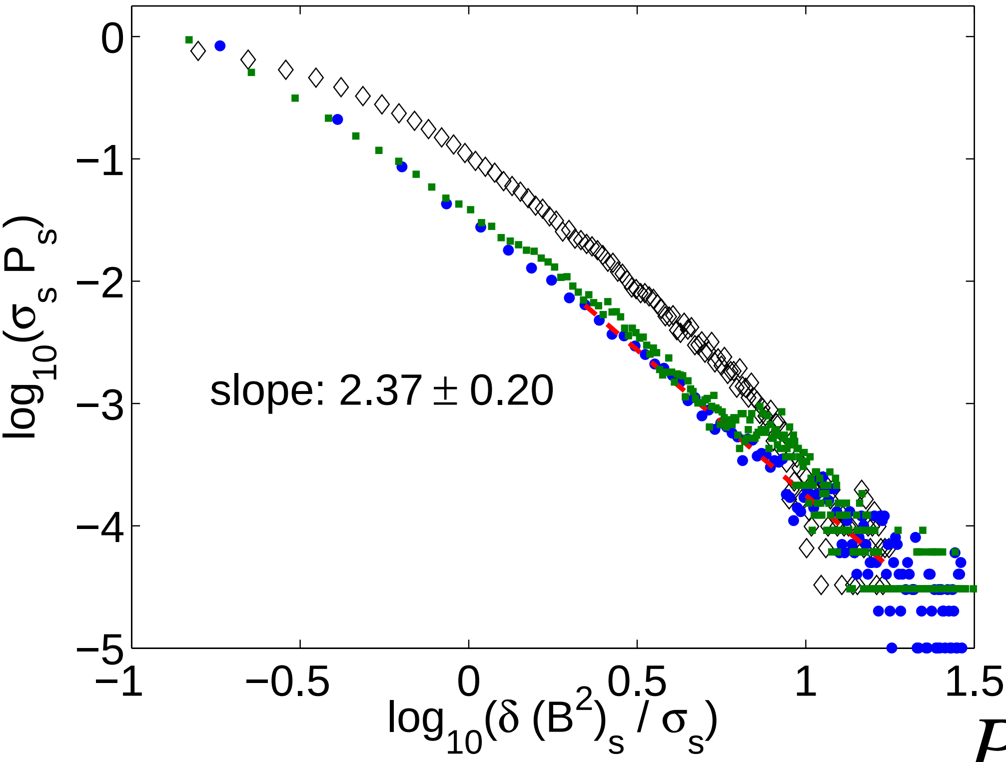


Scaling exponent:

H=0.44 ±0.02

Distribution functions

Fluctuations δB^2 at solar maximum are approximately self-similar, but at solar minimum they are not.



- Diamond – solar min
- Square – solar max, ACE
- Circle – solar max, WIND

At solar max the tails are a power low, consistent with a limiting form of a Levy

$$P_L(\left|\delta B^2\right| \to \infty) \propto \left|\delta B^2\right|^{-(1+\alpha)}$$

Stochastic Modelling

Self-similarity allows the application of stochastic methods developed for self-similar processes

Fractional Levy motion (correlations) [5]

$$0 < \alpha \leq 2, 0 \leq H \leq 1 \quad \partial_t P(x,t) = \alpha H t^{\alpha H - 1} \nabla^\alpha P(x,t)$$

Fokker-Planck description (small/no correlations) [6]

$$\frac{\partial P(x,t)}{\partial t} = \nabla[A(x)P + B(x)\nabla P]$$

[5] N.W. Watkins ***et.al.***, Phys. Rev. E 79 (2009).
[6] B. Hnat ***et al.***, Phys. Rev. E 67 (2003).

Dynamical model for fluctuations

$$\frac{\partial P(x,\tau)}{\partial \tau} = \nabla\left[A(x)P(x,\tau) + B(x)\nabla P(x,\tau)\right]$$

Transport coefficients: $A(x) \propto |x|^{1-1/\alpha}, B(x) \propto |x|^{2-1/\alpha}$

Change of variables: $P(x,\tau) = \tau^{-\alpha} P_s(x\tau^{-\alpha})$; solve for P_s

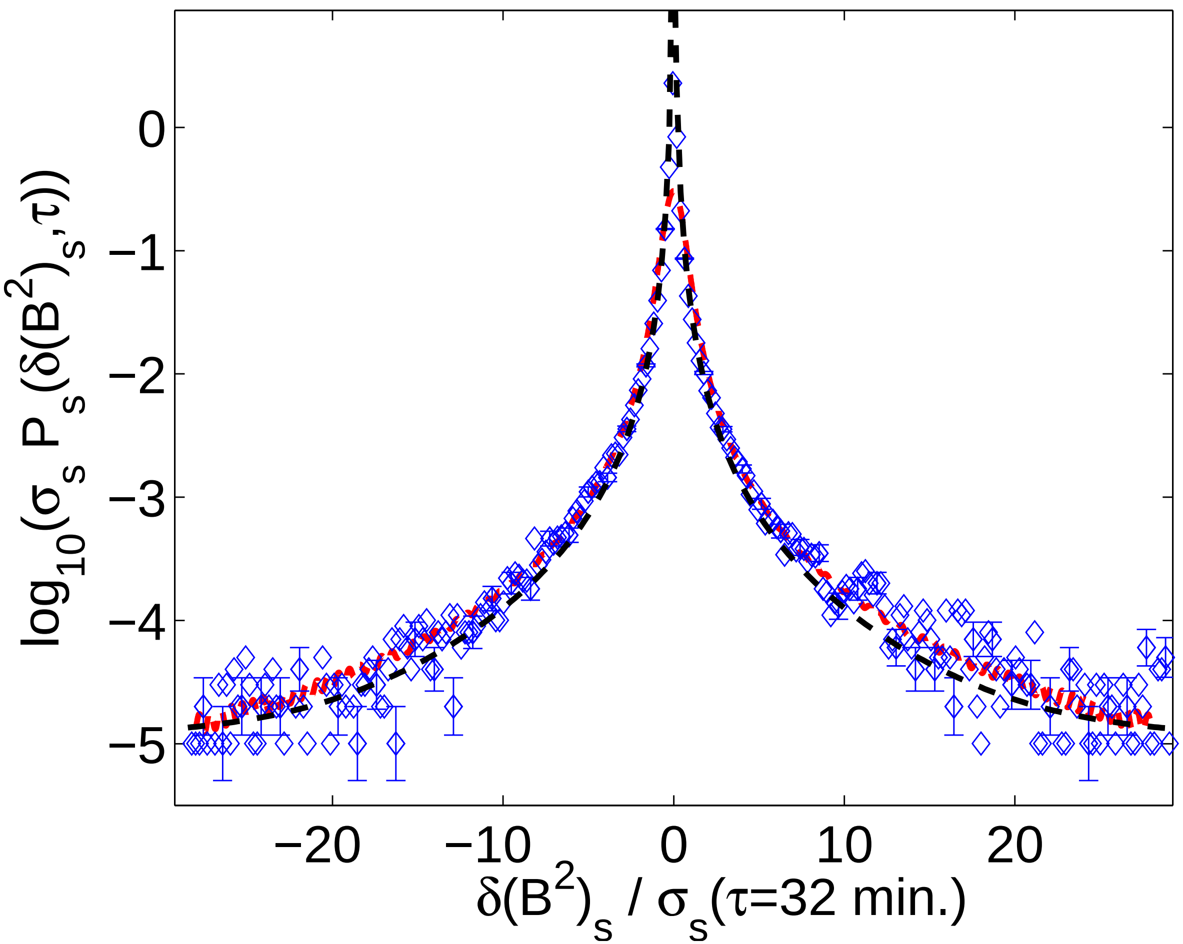
This corresponds to a Langevin equation: $\frac{dx}{dt} = \beta(x) + \gamma(x)\xi(t)$

and we can obtain β, γ via the Fokker- Planck coefficients

$$\left\langle \xi(\mathrm{t})\xi(\mathrm{t}+\tau) \right\rangle = \delta(\tau)$$

Distribution functions

Can fluctuations δB^2 be modelled by a Levy PDF?



Red: Levy PDF with α=1.4 and γ=0.3

$$P_L(x) = \int e^{-ikx} e^{-\gamma|k|^\alpha} dk$$

Black: Fokker-Planck

Conclusions

- Scaling of δB^2 at solar maximum appears to be well approximated by self-similarity

- Scaling is solar cycle dependent and is multi-fractal at solar minimum

- Self-similarity at solar maximum could be interpreted as a signature of complex fractal coronal magnetic configuration

- Self-similarity allows for application of well known stochastic modelling tools